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A Borsuk-Ulam Theorem for compact Lie group actions

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Abstract. Let G be a compact Lie group. Let X, Y be free G-spaces. In this paper, we consider the question of the existence of G-maps $f: X \to Y$. As a consequence, we obtain a theorem about the existence of $\mathbb{Z}_p$-coincidence points.

Keywords: Borsuk-Ulam Theorem, compact Lie group, free actions, equivariant maps.

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1 Introduction

One formulation of the Borsuk-Ulam Theorem [2] is that there is no map from S^m to S^n equivariant with respect to the antipodal map, when $m > n$ (see, for example, [1,7.2]). In [13], it was proved that if X and Y are Hausdorff, pathwise connected and paracompact spaces equipped with free involutions $T: X \to X$ and $S: Y \to Y$ such that for some natural $m \geq 1$, $H^q(X; \mathbb{Z}_2) = 0$ for $1 \leq q \leq m$ and $H^{m+1}(Y/S; \mathbb{Z}_2) = 0$, where Y/S is the orbit space of Y by S, then there is no equivariant map $f: (X, T) \to (Y, S)$. Our objective, in this paper, is to generalize this result for free actions of a compact Lie group G.

Let R be a PID and G a compact Lie group. Let X, Y be free G-spaces. We denote by $\beta_i(X; R)$ the i-th Betti number of X. Specifically, we prove

Theorem 1.1. *Let G be a compact Lie group and X, Y free G-spaces, Hausdorff, pathwise connected and paracompact. Suppose that for some natural $m \geq 1$, $H^q(X; R) = 0$ for $0 < q < m$ and $H^{m+1}(Y/G; R) = 0$. Then, if $\beta_m(X; R) < \beta_{m+1}(BG; R)$, there is no G-equivariant map $f: X \to Y$.*

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Let us observe that if Y is a topological manifold with a free action of a compact Lie group G, then $\dim(Y/G) = \dim(Y) - \dim(G)$, where dim denote the usual topological dimension. Thus, if $\dim(G) \geq 1$, one has that $\dim(Y/G) < \dim(Y)$. We have the following Corollary of Theorem 1.1.

Corollary 1.2. *Let G be a compact Lie group of dimension p. Let X be a free G-space, Hausdorff, pathwise connected and paracompact such that $H^q(X; R) = 0$, for $0 < q < m$ and let Y be a $(m+p)$-dimensional topological manifold with a free action of G. If $\beta_m(X; R) < \beta_{m+1}(BG; R)$, then there is no G-equivariant map $f : X \to Y$.*

Proof of Corollary 1.2. Since Y is a $(m+p)$-dimensional manifold with a free action of G, $\dim(Y/G) = m$ and therefore $H^{m+1}(Y/G; R) = 0$. It follows from Theorem 1.1 that there is no G-equivariant map $f : X \to Y$. □

The following examples illustrate Corollary 1.2.

Example 1.3. Let $R = \mathbb{Z}$, $G = S^1 \times S^1$, $X = S^5 \times S^5$ and $Y = S^3 \times S^3$, which admit free action of G. One has that $H^q(X; \mathbb{Z}) = 0$, for $0 < q < m = 5$ and $H^6(Y/G; \mathbb{Z}) = 0$, since $\dim(Y/G) = 4$. Moreover, $B(S^1 \times S^1) = \mathbb{C}P^\infty \times \mathbb{C}P^\infty$, which implies $\beta_5(X; \mathbb{Z}) = 2 < \beta_6(BG; \mathbb{Z}) = 4$. It follows from Corollary 1.2 that there is no G-equivariant map $f : X \to Y$.

Example 1.4. Let $R = \mathbb{Z}_2$, $G = \mathbb{Z}_2 \times S^1$, $X = S^4 \times S^5$, $Y = S^2 \times S^3$, which admit free action of G. One has that $H^q(X; \mathbb{Z}_2) = 0$, for $0 < q < m = 4$ and $H^5(Y/G; \mathbb{Z}_2) = 0$, since $\dim(Y/G) = 4$. Moreover, $B(\mathbb{Z}_2 \times S^1) = \mathbb{R}P^\infty \times \mathbb{C}P^\infty$, which implies $\beta_4(X; \mathbb{Z}_2) = 1 < \beta_5(BG; \mathbb{Z}_2) = 3$. It follows from Corollary 1.2 that there is no G-equivariant map $f : X \to Y$.

Remark 1.5. The referee pointed us that Example 1.3 can be obtained by using results in [9].

When $G = \mathbb{Z}_q$, where $q > 1$ is an integer, another consequence of Theorem 1.1 is the following

Corollary 1.6. *Let X, Y be free $\mathbb{Z}_q$-spaces, Hausdorff, pathwise connected and paracompact, where $q > 1$ is an integer. Let p be a prime which divides q. Suppose that $H^r(X; \mathbb{Z}_q) = 0$, for $1 \leq r \leq m$ and $H^{m+1}(Y/\mathbb{Z}_p; \mathbb{Z}_p) = 0$. Then there is no $\mathbb{Z}_q$-equivariant map $f : X \to Y$.*

Proof of Corollary 1.6. Since $\mathbb{Z}_p$ is a subgroup of $\mathbb{Z}_q$, we have that X, Y are free $\mathbb{Z}_p$-spaces. The hypothesis $H^r(X; \mathbb{Z}_q) = 0$, for $1 \leq r \leq m$ implies that $H^r(X; \mathbb{Z}_p) = 0$, for $1 \leq r \leq m$. In particular, $H^m(X; \mathbb{Z}_p) = 0$ implies that $\beta_m(X; \mathbb{Z}_p) < \beta_{m+1}(B\mathbb{Z}_p; \mathbb{Z}_p) = 1$. In this way, the assumptions of Theorem 1.1 are satisfied for $G = \mathbb{Z}_p$, then there is no $\mathbb{Z}_p$-equivariant map $f : X \to Y$. Consequently, there is no $\mathbb{Z}_q$-equivariant map $f : X \to Y$. □

Remark 1.7. Corollary 1.6 extends for free $\mathbb{Z}_q$-actions, $q > 2$, Theorem 1 proved in [13].

Remark 1.8. Suppose that in Corollary 1.6, Y is a m-dimensional manifold, thus $H^{m+1}(Y/\mathbb{Z}_p; \mathbb{Z}_p) = 0$. Then there is no $\mathbb{Z}_q$-equivariant map $f : X \to Y$. This particular case of Corollary 1.6 extends the following result proved by T. Kobayashi in [11, Theorem 1]: if X is a Hausdorff and pathwise connected space such that $H_r(X; \mathbb{Z}_q) = 0$, for $1 \leq r \leq m - 1$, then there is no $\mathbb{Z}_q$-equivariant map $f : X \to S^n$, where m, n are odd, $m > n$, S^n with the standard action of $\mathbb{Z}_q$, $q > 1$.

2 Preliminaries

We start by introducing some basic notions and notations. We assume that all spaces under consideration are Hausdorff and paracompact spaces. Throughout this paper, H_* and H^* will always denote the singular homology and cohomology groups. For a given space B, let $\mathcal{G}$ be a system of local coefficients for B. We will denote by $H_*(B; \mathcal{G})$ the homology groups of B with local coefficients in $\mathcal{G}$. The symbol $\cong$ will denote an appropriate isomorphism between algebraic objects.

Suppose that G is a compact Lie group which acts freely on a Hausdorff and paracompact space X, then $X \to X/G$ is a principal G-bundle [3, Theorem II.5.8] and one can take

$$h : \frac{X}{G} \to BG \tag{2.1}$$

a classifying map for the G-bundle $X \to X/G$.

Remark 2.1. Let us observe that if $\hat{h}$ is another classifying map for the principal G-bundle $X \to X/G$, then there is a homotopy between h and $\hat{h}$.

Given the G-space X, consider the product $EG \times X$ with the diagonal action given by $g(e, x) = (ge, gx)$ and let $EG \times_G X = (EG \times X)/G$ be its orbit space. The first projection $EG \times X \to EG$ induces a map

$$p_X \colon EG \times_G X \to \frac{(EG)}{G} = BG, \tag{2.2}$$

which is a fibration with fiber X and base space BG being the classifying space of G. This is called the *Borel construction*. It associates to each G-space X a space $EG \times_G X$, which will be denoted by X_G, over BG and to each G-map $X \to Y$ a fiber preserving map $EG \times_G X \to EG \times_G Y$ over BG.

Remark 2.2. If G acts freely on X, then the map

$$X_G \to \frac{X}{G} \tag{2.3}$$

induced by the second projection $EG \times X \to X$ is a fibration with a contractible fibre EG and therefore a homotopy equivalence (for details, see [7]).

Now, let us recall the following theorems of Leray-Serre for fibrations, as given in [12, theorems 5.1, 5.2].

Theorem 2.3 [The homology Leray-Serre Spectral Sequence]. *Let G be an abelian group. Given a fibration $F \hookrightarrow E \xrightarrow{p} B$, where B is pathwise connected, there is a first quadrant spectral sequence $\{E^r_{*,*}, d^r\}$, with*

$$E^2_{p,q} \cong H_p(B; \mathcal{H}_q(F; G)), \tag{2.4}$$

the homology of B with local coefficients in the homology of F, the fibre of p, and coverging to $H_*(E; G)$. Furthermore, this spectral sequence is natural with respect to fibre-preserving maps of fibrations.

Theorem 2.4 [The cohomology Leray-Serre Spectral Sequence]. *Let R be a commutative ring with unit. Given a fibration $F \hookrightarrow E \xrightarrow{p} B$ where B is pathwise connected, there is a first quadrant spectral sequence of algebras $\{E_r^{*,*}, d_r\}$, with*

$$E_2^{p,q} \cong H^p(B; \mathcal{H}^q(F; R)), \tag{2.5}$$

the cohomology of B with local coefficients in the cohomology of F, the fibre of p, and coverging to $H^*(E; R)$ as an algebra. Furthermore, this spectral sequence is natural with respect to fibre-preserving maps of fibrations.

3 Proof of Theorem 1.1

The proof of Theorem 1.1 will follow from the following lemmas

Lemma 3.1. *Let R be a PID and $E \xrightarrow{p} B$ a fibration with fiber F and base space B pathwise connected. Suppose that $H_q(F, R) = 0$, for $0 < q < m$. Then, there exists an exact sequence with coefficients in R,*

$$H_{m+1}(E) \xrightarrow{p_*} H_{m+1}(B) \xrightarrow{\tau} H_0(B, \mathcal{H}_m(F)) \to H_m(E) \xrightarrow{p_*} H_m(B) \to \cdots$$
$$\cdots \to H_2(E) \xrightarrow{p_*} H_2(B) \xrightarrow{\tau} H_0(B, \mathcal{H}_1(F)) \to H_1(E) \xrightarrow{p_*} H_1(B) \to 0,$$

where τ is the transgression homomorphism and $\mathcal{H}_i(F)$ denotes the system of local coefficients over B.

Proof. It follows from Theorem 2.3 that there exists a first quadrant spectral sequence $\{E^r_{*,*}, d^r\}$, with

$$E^2_{p,q} \cong H_p(B; \mathcal{H}_q(F)), \tag{3.1}$$

the homology of B with local coefficients in the homology of F, the fibre of p, and coverging to $H_*(E; R)$. Since F is pathwise connected the local coefficients system $\mathcal{H}_0(F)$ over B is trivial and follows from [12, Proposition 5.18] that

$$E^2_{p,0} \cong H_p(B; \mathcal{H}_0(F)) = H_p(B; H_0(F)) = H_p(B), \quad \forall p. \tag{3.2}$$

On the other hand, $H_q(F) = 0$, for $0 < q < m$ and follows from (3.1) that $E^2_{p,q} = E^\infty_{p,q} = 0$, for $0 < q < m$. Furthermore, the spectral sequence is a first quadrant spectral sequence, then we have

$$H_{m+1}(B) = E^2_{m+1,0} = E^3_{m+1,0} = \cdots = E^{m+1}_{m+1,0} \tag{3.3}$$
$$H_0(B; \mathcal{H}_m(F)) = E^2_{0,m} = E^3_{0,m} = \cdots = E^{m+1}_{0,m} \tag{3.4}$$
$$H_p(B) = E^2_{p,0} = E^3_{p,0} = \cdots = E^p_{p,0} = E^\infty_{p,0}, \quad \forall p \leq m. \tag{3.5}$$

Consider the following exact sequences

$$0 \to E^\infty_{0,r} \to H_r(E) \to E^\infty_{r,0} \to 0 \tag{3.6}$$
$$0 \to E^\infty_{r,0} \to E^r_{r,0} \xrightarrow{d^r} E^r_{0,r-1} \to E^\infty_{0,r-1} \to 0, \tag{3.7}$$

for any $r \leq m+1$. Putting together these sequences, one obtains the exact sequence

$$H_{m+1}(E) \to E^{m+1}_{m+1,0} \stackrel{d^{m+1}}{\to} E^{m+1}_{0,m} \to H_m(E) \to E^m_{m,0} \stackrel{d^m}{\to} E^m_{0,m-1} \to \cdots$$
$$\cdots \to H_2(E) \to E^2_{2,0} \stackrel{d^2}{\to} E^2_{0,1} \to H_1(E) \to E^\infty_{1,0} \to 0$$

where $d^r : E^r_{r,0} \to E^r_{0,r-1}$ is the transgression homomorphism [12, theorem 6.5]. If we replace in (3.8) the equalities (3.3), (3.4) and (3.5), one obtains the desired sequence, that is,

$$H_{m+1}(E) \stackrel{p_*}{\to} H_{m+1}(B) \stackrel{\tau}{\to} H_0(B, \mathcal{H}_m(F)) \to H_m(E) \stackrel{p_*}{\to} H_m(B) \to \cdots$$
$$\to \cdots H_2(E) \stackrel{p_*}{\to} H_2(B) \stackrel{\tau}{\to} H_0(B, \mathcal{H}_1(F)) \to H_1(E) \stackrel{p_*}{\to} H_1(B) \to 0$$

This completes the proof. □

Lemma 3.2. *Let R be a PID and $E \stackrel{p}{\to} B$ a fibration with fiber F and base space B pathwise connected. Suppose that $H^q(F, R) = 0$, for $0 < q < m$. Then, there exists an exact sequence with coefficients in R,*

$$0 \to H^1(B) \stackrel{p^*}{\to} H^1(E) \to H^0(B; \mathcal{H}^1(F)) \stackrel{\tau}{\to} H^2(B) \stackrel{p^*}{\to} H^2(E) \to \cdots$$
$$\to \cdots H^m(B) \stackrel{p^*}{\to} H^m(E) \to H^0(B; \mathcal{H}^m(F)) \stackrel{\tau}{\to} H^{m+1}(B) \stackrel{p^*}{\to} H^{m+1}(E)$$

where τ is the transgression homomorphism and $\mathcal{H}^i(F)$ denotes the system of local coefficients over B.

Proof. The proof is analogous to Lemma 3.1, considering the cohomology Leray-Serre Spectral Sequence (Theorem 2.4) associated to the fibration $E \stackrel{p}{\to} B$. □

Lemma 3.3. *Let X be a free G-space, Hausdorff, pathwise connected and paracompact. For a natural number $m \geq 1$, suppose that $H^q(X; R) = 0$, for $0 < q < m$ and that $\beta_m(X; R) < \beta_{m+1}(BG; R)$. Then the homomorphism $h^* \colon H^{m+1}(BG; R) \to H^{m+1}(X/G; R)$ is nontrivial, where $h \colon X/G \to BG$ is a classifying map for the principal G-bundle $X \to X/G$.*

Proof. Let $EG \to BG$ be the universal G-bundle and $h\colon X/G \to BG$ a classifying map for the principal G-bundle $X \to X/G$. Let $p_X\colon X_G \to BG$ the Borel-fibration associated to the G-space X, where X_G is the Borel space, as in (2.2). It follows from Remark 2.2 that the map $X_G \to X/G$ is a homotopy equivalence. Let $r\colon X/G \to X_G$ be its homotopy inverse. Then $p_X \circ r\colon X/G \to BG$ also classifies the principal G-bundle $X \to X/G$, and it follows from Remark 2.1 that the map $(p_X \circ r)$ is homotopic to h. Since

$$r^*\colon H^{m+1}(X_G; R) \to H^{m+1}\left(\frac{X}{G}; R\right)$$

is an isomorphism, it suffices to prove that $p_X^*\colon H^{m+1}(BG; R) \to H^{m+1}(X_G; R)$ is nontrivial. In fact, since $H^q(X; R) = 0$, for $0 < q < m$, it follows from Lemma 3.2 that there exists an exact sequence with coefficients in R,

$$\begin{gathered} 0 \to \cdots \to H^m(X_G) \to \cdots \\ \cdots \to H^0(BG; \mathcal{H}^m(X)) \xrightarrow{\tau} H^{m+1}(BG) \xrightarrow{p_X^*} H^{m+1}(X_G) \end{gathered} \tag{3.9}$$

Suppose that $p_X^*\colon H^{m+1}(BG; R) \to H^{m+1}(X_G; R)$ is the zero homomorphism. From (3.9), we have that $\tau\colon H^0(BG; \mathcal{H}^m(X)) \to H^{m+1}(BG)$ is a surjective homomorphism, which implies that

$$\text{rank } H^0(BG; \mathcal{H}^m(X)) \geq \beta_{m+1}(BG; R). \tag{3.10}$$

On the other hand, since $H^0(BG; \mathcal{H}^m(X))$ is isomorphic to a submodule of $H^m(X; R)$ [14, theorem 3.2] and by hypothesis $\beta_m(X; R) < \beta_{m+1}(BG; R)$,

$$\text{rank } H^0(BG; \mathcal{H}^m(X)) < \text{ rank } H^m(X; R) = \beta_m(X; R) < \beta_{m+1}(BG; R),$$

which contradicts 3.10. □

Remark 3.4. A similar result to Lemma 3.1 has been proved in [10, Lemma 2], when G is a finite group.

Proof of Theorem 1.1. Suppose that $f\colon X \to Y$ is a G-equivariant map. Since Y is a Hausdorff paracompact space, one can take a classifying map $g\colon Y/G \to BG$ for the principal G-bundle $Y \to Y/G$. Then the map $h = g \circ \bar{f}\colon X/G \to BG$ can be taken as a classifying map for the principal G-bundle $X \to X/G$, where $\bar{f}\colon X/G \to Y/G$ is the map induced by f between the orbit spaces. Since by hypothesis $H^{m+1}(Y/G; R) = 0$

one has that $g^*\colon H^{m+1}(BG; R) \to H^{m+1}(Y/G; R)$ is trivial and consequently $h^*\colon H^{m+1}(BG; R) \to H^{m+1}(X/G; R)$ is the zero homomorphism, which contradicts Lemma 3.3. □

Suppose X equipped with a free action of the cyclic group $\mathbb{Z}_p$ generated by a periodic homeomorphism $T : X \to X$ of period p, where p is a prime. We set $Y^* = \Pi_{i=1}^{p} Y^i - \Delta$, where

$$\Delta = \{(y_1, y_2, \cdots, y_p) \in \Pi_{i=1}^{p} Y^i; y_1 = y_2 = \cdots = y_p)\}$$

is the usual diagonal in $\Pi_{i=1}^{p} Y^i$. Then, Y^* admits a free action of $\mathbb{Z}_p$, generated by a periodic homeomorphism $t_Y : Y^* \to Y^*$ of period p given by

$$t_Y(y_1, y_2, \cdots, y_p) = (y_2, y_3, \cdots, y_p, y_1).$$

Under these conditions, we obtain the following

Theorem 3.5. *For a natural number $m \geq 1$, suppose that $H^r(X; \mathbb{Z}_p) = 0$, for $1 \leq r \leq m$ and that $H^{m+1}(Y^*/t_Y; \mathbb{Z}_p) = 0$, where p is a prime. Then every continuous map $f : X \to Y$ has a $\mathbb{Z}_p$-coincidence, that is, there exists a point $x \in X$ such that $f(x) = f(gx)$ for any $g \in \mathbb{Z}_p$.*

Proof. Let $f : X \to Y$ be a map without $\mathbb{Z}_p$-coincidences. Then we can define the $\mathbb{Z}_p$-equivariant map $F : X \to Y^*$ by

$$F(x) = (f(x), f(T(x)), \cdots, f(T^{p-1}(x)).$$

The existence of such a map contradicts Corollary 1.6. □

Remark 3.6. Let us observe that Theorem 3.5 extends for free $\mathbb{Z}_p$-actions, $p > 2$, Theorem 3 proved in [13].

Remark 3.7. Suppose that in Theorem 3.5, Y is a finite connected k-dimensional CW-complex. Then $Y^*/\mathbb{Z}_p$ admits a pk-dimensional structure of a CW-complex, thus, $H^{pk+1}(Y^*/\mathbb{Z}_p, \mathbb{Z}_p) = 0$. Then, every continuous map $f : X \to Y$ has a $\mathbb{Z}_p$-coincidence, if $n > pk$ (this also follows from Theorem 1 of [10]; in fact, Theorem 1 of [10] gives additionally that the result is also valid for $n = pk$).

Remark 3.8. Let X be a Hausdorff space which supports a free $\mathbb{Z}_p$-action, where $p \geq 2$ is a prime. In [4], F. Cohen and J.E. Connett obtained a Borsuk-Ulam result for continuous maps $f : X \to \mathbb{R}^n$, with $n \geq 2$. The following statement was proved: if X is $(n-1)(p-1)$-connected, then there exist $x \in X$ and $g \in \mathbb{Z}_p$, $g \neq$ identity, such that $f(x) = f(g(x))$. In the following Theorem, we replace the hypothesis "X is $(n-1)(p-1)$-connected" by a cohomological condition on X.

Theorem 3.9. *Let X be a Hausdorff, pathwise connected and paracompact space, equipped with a free action of the cyclic group $\mathbb{Z}_p$ generated by a periodic homeomorphism $T : X \to X$ of period p, where p is a prime. Suppose that $H^r(X, \mathbb{Z}_p) = 0$, for $1 \leq r \leq (n-1)(p-1)$. Then for every continuous map $f : X \to \mathbb{R}^n$, there exists $x \in X$ and $1 \leq i \leq p-1$ such that $f(x) = f(T^i(x))$.*

Remark 3.10. It is interesting to note that Theorem 3.9 is stronger than the result proved in [4], since a $(n-1)(p-1)$-connected space has $H^r(X, \mathbb{Z}_p)$ equal to zero for $1 \leq r \leq (n-1)(p-1)$.

To prove Theorem 3.9, we recall the definition of the *configuration space* of a manifold M, studied by Fadell and Neuwirth [8] in 1962. The ordered *configuration space* is the space of the all ordered k-tuples of distinct points in M defined by

$$F(M, k) = \{\langle x_1, x_2, \cdots, x_k\rangle \in M^k : x_i \neq x_j, \quad \text{for all} \quad i \neq j\}. \tag{3.11}$$

When $M = \mathbb{R}^n$, the space $F(\mathbb{R}^n, k)$ is the complement of a linear arrangement of subspaces of codimension n in $\mathbb{R}^{kn}$. The cohomology of these spaces was obtained by Cohen [5, 6]. It is again torsion free, with generators of degree $(n-1)$ corresponding to individual subspaces, and relations corresponding to triples with the same pairwise intersection.

The symmetric group $\sum_k$ on k letters acts freely on $F(M, k)$ by permutation of coordinates. If G is any finite group, there exists an integer k such that $G \subset \sum_k$. Thus $\mathbb{Z}_p$, the cyclic group of order p, acts freely on $F(\mathbb{R}^n, p)$, via the action given by a homomorphism $\mathbb{Z}_p \to \sum_p$ which sends $1 \in \mathbb{Z}_p$ to the cycle $(1, 2, \cdots, p)$.

In [4], Cohen and Connett proved the following result

Lemma 3.11. $H^r(F(\mathbb{R}^n, p)/\mathbb{Z}_p; \mathbb{Z}_p) = 0$, *for* $r > (n-1)(p-1)$.

Proof of Theorem 3.9. Suppose that $f(x) \neq f(T^i(x))$, for any $x \in X$ and $1 \leq i \leq p-1$. Thus, we can define a $\mathbb{Z}_p$-map $F: X \to F(\mathbb{R}^n, p)$ given by

$$F(x) = \langle f(x), f(Tx), \cdots, f(T^{p-1}x) \rangle.$$

Since $H^r(X, \mathbb{Z}_p) = 0$, for $1 \leq r \leq (n-1)(p-1)$ and by Lemma 3.11, $H^r(F(\mathbb{R}^n, p)/\mathbb{Z}_p; \mathbb{Z}_p)$ is zero, for $r > (n-1)(p-1)$, one has that X and $F(\mathbb{R}^n, p)$ satisfy the hypotheses of Corollary 1.6 and the existence of such a $\mathbb{Z}_p$-equivariant map is a contradiction.

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